Alice’s Multiplication Tables

“... Oh dear, how puzzling it all is! I'll try if I know all the things I used to know. Let me see: four times five is twelve, and four times six is thirteen, and four times seven is—oh dear! I shall never get to twenty at that rate.

Why couldn't Alice get to twenty at that rate?

Hint: This problem involves different ways of counting; for example, we count weeks using base 7. Ten days becomes one week and three days; 10 is denoted as 13 in base 7. Fifteen days would be two weeks and one day remaining; 15 is denoted as 21 in base 7. If we count in a base larger than ten, we need to invent new symbols for some numbers; twenty-six is 1T in base 16, where T stands for ten.

Now let us go back to Alice’s multiplication tables:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>4 x 5 = 12</td>
<td>20 counting in base 18</td>
</tr>
<tr>
<td>4 x 6 = 13</td>
<td>24 counting in base 21</td>
</tr>
<tr>
<td>4 x 7 = ...</td>
<td>(continue the pattern)</td>
</tr>
</tbody>
</table>

If you extend the multiplication tables far enough, you will discover why Alice could not reach an answer of 20.

Answer on p. 8.

See p. 5

In Chapter 11 of A Study in Scarlet, we read that [Sherlock] Holmes once wrote an article entitled “The Book of Life,” in which he claimed that the conclusions of one trained to observation and analysis would be “as infallible as so many propositions of Euclid. So startling would his results appear to the uninitiated that until they learned the processes by which he had arrived at them they would well consider him a necromancer.”


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